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SOME GENERAL NOTES ON OCEAN WAVES AND WAVE FORCE.*

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WITH DISCUSSION.

A wave in a general sense has been defined as a state of disturbance propagated from one part of a medium to another. Through the oscillatory action of the particles of the medium, the energy due to a disturbance is transferred from one point to another. There is not, necessarily, a transfer of matter through the action of a wave, though in some cases the wave displaces, usually to a small amount only, the medium through which it passes. Currents, on the other hand, imply the passage of matter associated with energy.

Waves may be free or forced. A free wave is one which is no longer acted upon by the disturbing influence, either through the cessation of the disturbance or through the wave having outrun the sphere of disturbance. A forced wave is one upon which the disturbing influence continues to act so as to modify its propagation.

* A large portion of the following article has been collated from various sources at different times, and is not claimed by the author as original.

Thus, while a gale is blowing the character of the water wave is continually being modified; beyond the immediate influence of a gale, or after it subsides, regularly oscillating waves or rollers are obtained, for the long ones not only outstrip the shorter ones, but are less speedily worn down by fluid friction.

The ocean tide is mainly a forced wave, depending on the continued action of the moon and sun. The tidal wave in an estuary or river is practically free, being almost independent of moon and sun, and depending mainly upon the configuration of the channel and the tidal disturbance at the mouth.

In the Thames, the mean range of the tide at Sheerness is about 13 ft.; at Deptford, about 17; at London Bridge, about 15; and from this point it diminishes gradually to the weir at Teddington where it is 2 ft.

At the entrance of Bristol Channel the whole rise at spring tides is about 18 ft.; at Swansea, about 30 ft., and at Chepston, about 50 ft.

At the entrance of the Bay of Fundy, the currents are very rapid, but the range is only about 8 ft.; at the head of the bay, where it branches into two shoal arms, the range of the tide sometimes amounts to 60 to 70 ft.

At Cherbourg, it is only 15 ft., but at St. Malo it rises nearly to 40 ft.

The most usual division of free waves is into long waves, oscillatory waves and ripples. The first two classes run by gravity and the third mainly by surface tension. Long waves, or waves in water that is very shallow, compared with the length of the wave, are due to disturbances which extend to the full depth of the water. Oscillatory waves are due to disturbances confined to the upper layers of the water, which die away with great rapidity in successive layers below.

Mr. Scott Russell called the long wave a "wave of translation," in contradistinction to a "wave of oscillation." Under waves of translation he included the tidal wave, bores,* ordinary ground swell and waves

* Airy says: "We believe the following description of the cause and appearance of the bore will be found correct. It is necessary for its formation that there be a very large tide rising with great rapidity (thus at Newnham, where the water rises 18 ft. in an hour and a half, the bore is considerable). It is necessary also that the channel of the river be bordered with a great extent of flat sands near to the level of low water. These circumstances hold in the Severn, the Seine, the Amazon, the bays at the head of the Bay of Fundy (Chignecto Bay and Bay of Mines), and other places where the bore is remarkable. When the rise of the tide begins, the surface of the water is disturbed in mid-channel, but the water is not broken, it is merely a common wave. But as this rapid rise elevates the surface suddenly above the level of the flat sands the water immediately rushes over them with great velocity and a broken front, making a great noise. This is the whole of the bore; it is, however, a majestic phenomenon."

of oscillation when they reached much shallow water. Under waves of oscillation he included all other storm waves.

This classification has been adopted by many subsequent writers. This division, however, has no value as a classification, and is very apt to induce erroneous ideas on the action and power of waves. It is recognized now that all storm waves are waves of translation to a more or less extent, according to the strength of the wind, the time it has been blowing, and other circumstances.

Mr. Scott Russell, in describing his wave of translation, says:

"As the wave advances the particles of water of which it is composed, instead of continuing to travel forward themselves, force other particles upwards into the wave and themselves subside and take the place of these latter particles. These again behave themselves similarly, depositing themselves in the place of further particles; and in this manner the wave proceeds constant in shape and velocity but continually recomposed of new particles of water which are successively lifted up, carried forward and let down again into new positions.

"A solitary wave of elevation obviously carries across any fixed transverse section a quantity of water equal to that which lies above the undisturbed level. If the wave had been one of depression, the translation would have been in the opposite direction to that of the wave motion. Hence, when the wave consists of an elevation, followed by a depression of equal volume, it leaves the water as it found it."

The theory of motion of such a wave is based on the hypothesis that all the particles in a transverse section have at the same instant the same horizontal speed, the vertical motion of the water depending on the change of horizontal speed from section to section. The speed of propagation of such a wave of translation is given by Mr. Scott Russell as that which would be acquired by a body falling through half the depth of the water,

$$V = \sqrt{g\,h}$$

h being the depth of water measured from the wave crest.

Rankine states that long waves or waves in water that is very shallow compared with the length of the wave have a velocity nearly independent of the wave length and nearly equal to that acquired by a body falling through a height equal to half the depth of the water added to three-fourths of the height of the wave.

$$V = \sqrt{g\,(h^1 + \tfrac{3}{4}\,D)},$$

h^1 being depth of still water and D the wave height.

These two formulas are identical, for the wave crest is about three-fourths of the depth of the wave above still water, so $h = h^1 + \tfrac{3}{4}\,D$.

The tidal wave travels at different speeds in different seas and oceans, dependent upon the depths of the sea. In the North Sea it travels 45 miles per hour. The Atlantic tidal wave (long wave) passes over 90° of latitude in 12 hours, or at a rate of speed of 520 miles per hour. The great earthquake wave of 1868 was estimated to have traveled in the Pacific Ocean at a rate of 300 to 400 miles per hour. The great wave formed in the Straits of Sunda by the volcanic eruption of Krakatoa was felt upon the coast of France, and its speed estimated at over 300 miles per hour.

The nearest example to a typical oscillatory wave is found in what is called the swell, or regular rolling waves, which continue to run in deep water after a storm. In a pure oscillatory wave, the motion of each particle is in nearly a circular orbit; the particles at the surface of the water describe the longest orbits; the extent of the motion, both horizontal and vertical, diminishes in a geometrical progression as the depths increase in arithmetical progression and at great depths the motion is insensible.

Fig. 1 is Weber's diagram of an oscillatory wave, Fig. 2 showing the same filaments in still water, *O O* being the level of the surface in still water. The particles of water are rotating to the right in circles whose radii vary from r equal R for the surface particles, to r^1 equals $r\,e^{-\frac{k}{r}}$ for all particles at a lower depth, k being the depth of the filament below still water, e the base of the Naperian logarithms, r and R as given later, and $-\frac{k}{r}$ being an exponent to e.

The trachoidal lines *O*, *a*, *b*, *c*, etc., in Fig. 1, represent the form taken by the filaments *O*, *a*, *b*, *c*, etc., in Fig. 2 during the passage of a wave. The left-hand portion of Fig. 1 shows the distortion of the filaments, both horizontally and vertically, this being greatest at the surface and diminishing rapidly as the depth increases. The wave is traveling to the right, or in the same direction as the particle at the top of its orbit. The energy imparted to the particles is expended in lifting the particle the distance of its center of rotation above its position at still water, and in giving it rotation about the center. It can be shown that one-half of the energy of disturbance goes to lift the particle to its mean height and one-half to impart rotation.

Figs. 3 and 4 have been prepared to show the form of wave generated when the particles travel in elliptical orbits. Both the waves

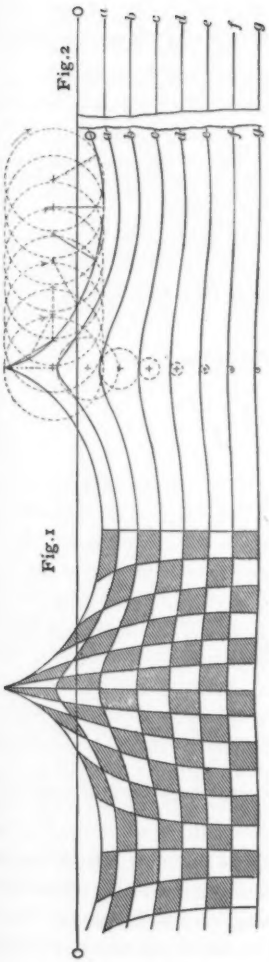


Fig.1

Fig.2

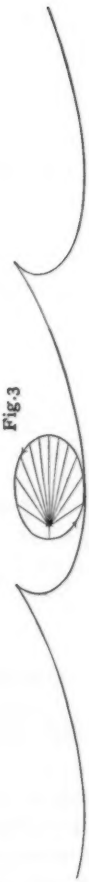


Fig.3

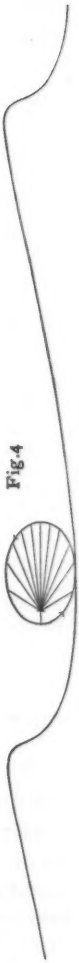


Fig.4

shown are generated by the same ellipse, but the wave length in Fig. 4 is made double that of Fig. 3.

The following formulas (condensed by Prof. C. S. Lyman) are those giving the relations between the elements of such an oscillatory wave:

l = length of wave from crest to crest.
D = depth from trough to crest.
V = velocity of propagation of the wave.
t = period or interval of time between consecutive wave crests.
r = radius of orbit of a particle at the surface.
v = velocity of a particle on its orbit or at the crest of a wave.
R = radius of circle whose circumference is a wave length, also the length of a pendulum keeping time with the wave.
a = elevation of crest above still water.
b = depression of trough below still water.
g = acceleration of gravity.

$$l = t\,V = 2\,\pi\,R = \frac{g\,t^2}{2\,\pi} = \frac{2\,\pi\,V^2}{g}.$$

$$D = 2r.$$

$$t = \sqrt{\frac{2\,\pi\,l}{g}} = 2\,\pi\sqrt{\frac{R}{g}} = \frac{l}{V}.$$

$$V = \frac{l}{t} = \frac{g\,t}{2\,\pi} = \sqrt{\frac{g\,l}{2\,\pi}}^{*} = \sqrt{g\,R} = \frac{g\,r}{v}.$$

$$v = \frac{2\,\pi\,r}{t} = r\sqrt{\frac{g}{R}} = r\sqrt{\frac{2\,\pi\,g}{l}} = \frac{V\,r}{R} = \frac{2\,\pi\,r\,V}{l} = \frac{g\,r}{V} = \frac{t\,g\,r}{l}.$$

$$a = r + \frac{\pi\,r^2}{l} = r + \frac{r^2}{2R}\,;\; b = r - \frac{\pi\,r^2}{l} = r - \frac{r^2}{2\,R}.$$

Sine of angle of steepest slope is sin. $\theta = \frac{2\,\pi\,r}{l} = \frac{r}{R}$.

$R : r = g : \frac{4\,\pi^2\,r}{t^2}$, or as gravity is to the centrifugal force of a surface particle.

The maximum height of a wave depends on the centrifugal force of the particles, and thus on the external forces generating it. When the centrifugal force becomes equal to gravity, or $r = R$, the resultant force for a particle at the highest point of its orbit or at the crest of the

* $V = \sqrt{\frac{g\,l}{2\,\pi}}$ is the velocity due to a heavy body falling $\frac{l}{4\,\pi}$ or as it is sometimes worded, approximately one-thirteenth of the wave length.

wave must be zero. Beyond this point the particle flies from its orbit and the crest breaks into foam.

The greatest elevation of the crests above the level of still water is also when $r = R$ and equals three-fourths of the depth of the wave, and the centers of the orbits of the surface particles will be on a line one-fourth of the depth of the wave above the level of still water.

Capt. Gaillard, from 45 observations on waves 2½ to 6 ft. deep just before breaking, found that the elevation above still water varied from 0.67 to 0.89, with a mean of 0.76; with a flat slope of the bottom and opposing wind it was increased, while with a steep slope and favorable wind it was decreased.

Stevenson says waves at Wick's Bay were two-thirds above and one-third below mean level.

Rankine gives elevation of crest above still water $= \frac{D}{2} + 0.7854 \frac{D^2}{l}$

depression of trough below still water $= \frac{D}{2} - 0.7854 \frac{D^2}{l}$

The height of the wave is, within certain limits, independent of its length.

The longer waves travel the faster.

The normal oscillatory wave to which the preceding theoretical equations apply is the wave on deep water or water more than a wave length in depth. In shallower water the orbits are no longer circles, but approximately ellipses of less height than length according to the degree of shallowness.

When a series of waves advances into water gradually becoming shallower, their periods remain unchanged, but their speed, and consequently their length, diminishes and their slopes become steeper. The orbits become distorted in such a manner that the front of the wave becomes steeper than the back, the crest, as it were, advancing faster than the troughs. At length the front of the wave curls over beyond the vertical, its crest falls forward and the wave breaks.

Purely oscillatory waves rarely, if ever, occur in the ocean. They are also more or less waves of translation. The forward horizontal motion of each particle under the crest is not quite compensated for by its backward movement under the trough, so that the wave sets down each particle of water a little in advance of where it picked that

particle up, causing what is known to sailors as the "heave of the sea." This effect, however, like the whole disturbance, is greatest at the surface layer and diminishes rapidly for each lower layer.

Long waves are analogous to oscillatory waves with the modification that, owing to the enormous length of the waves as compared with the depth of the water, the extent of horizontal motion of the particles is nearly equal at all depths and the extent of the vertical motion in any layer is nearly in the simple proportion of its height above the bottom. The orbit of such particles is a very long and flat ellipse. The deeper a particle is situated, the flatter becomes the orbit, a particle in contact with the bottom moving backward and forward in a horizontal line.

Comparing the equations for the speed of propagation of a long wave, $V = \sqrt{g\,h}$, with that for an oscillatory wave, $V = \sqrt{\frac{g\,l}{2\,\pi}} = \sqrt{g\,R}$, it will be seen that they are very similar in form.

Rankine gives as a general equation for the velocity of waves of all figures and for every kind of disturbance, provided only that the upper surface of the water is free:

$$V = \sqrt{g\,k}$$

k being the virtual depth of uniform disturbance, or, more briefly, the virtual depth, or a depth which, with a uniform velocity equal to the horizontal surface velocity of a particle, would give the same volume of horizontal displacement as occurs at a vertical plane through the crest of a wave with the varying velocities of any wave.

For an oscillatory wave, $k = R$, and for a long wave, $k = h$.

When oscillatory waves pass from deep to shallower water, imparting their energy to a less and less mass of water, and flattening their orbits, they take on more and more the characteristics of long waves. Mr. Scott Russell says that if waves be propagated in a channel whose depth diminishes uniformly, the waves will break when their height becomes equal to the depth of the water. He has never seen a wave of 10 ft. in 10 ft. of water, nor so much as 20 ft. in 20 ft. of water, nor 30 ft. high in 5 fathoms of water, but has seen them approach near these limits.

M. Bazin has shown that a wave breaks when its height exceeds two-thirds of the total depth. The most trustworthy measurements of waves in the mid-ocean show 44 to 48 ft. to be a remarkable height.

Waves having a height greater than 30 ft. are not commonly encountered. Scoresby measured waves 43 ft. high, 559 ft. long, traveling at a velocity of 32½ miles per hour, at 16-second intervals.

Dr. Schott (1891–92) reports the greatest storm wave observed by him as 36 ft. above sea level. Lieut. Paris records 37 feet above sea level. Dr. Schott says the heaviest storm waves he met moved about 58 ft. per second, and attained a length of about 650 ft. He confirms Lieut. Paris' determinations that the proportion between wave height and wave length in a high sea is as 1 to 18, and in a moderate sea as 1 to 33. Capt. Gaillard, from 58 observations at St. Augustine on waves varying from 4 ins. to 6 ft. in height, found that the depth at which waves broke varied from 0.72 to 2.0 times the wave height, but that for the greater number, it equaled the wave height. A strong, favorable wind made it equal 1.25, while an equal wind in the contrary direction caused it to fall to 0.72; when there was no wind and the bottom was uniform and sloping only 1 in 100, the depth was nearly always 1, but when the slope was about 1 in 12, it became 2. Officers of the United States Navy have recorded waves measuring ½ mile from crest to crest and a period of 23 seconds.

Waves having lengths of 500 to 600 ft. and periods of 10 to 11 seconds, are the ordinary storm waves of the Northern Atlantic.

Mr. Deverell observed in the equatorial calms a swell, which was never absent and measured 4 ft. high, 500 ft. long, and had periods of 10 seconds.

The study of waves and wave action is one of great importance to engineers.

The strength, modeling and stability of vessels, the best speed of boats in channels of limited cross-section, the wash of banks in canals and impounding reservoirs, the protection of sea coasts, the improvement and maintenance of channels and harbors, the design and maintenance of artificial structures which are liable to wave action, whether they be inland and subject to the sudden onslaught of a wave produced by the rupture of some restraining medium, or located on the coast and subject to the attack of sea waves—all require a knowledge of water waves and their action.

This paper will be confined to the consideration of the action of waves or the force of the sea upon artificial structures.

Smeaton classed the action of the sea as among the powers of nature

that are not subject to calculation. Since his time, experience with sea structures and knowledge of the laws governing the force of waves have been greatly extended. The valuable collection of facts and experiments by Mr. Thomas Stevenson and others upon the power exerted by sea waves, and the study of the laws governing the action and forms of waves by Messrs. Scott Russell, Rankine, Stokes and many other able investigators, go far to enable a reasonable estimate to be made of the forces to be encountered by structures subject to wave action.

Mr. Scott Russell says:

"In sea works there were practically two classes of waves to deal with. In deep water there were not only the oscillatory surface waves to be encountered, but also those which he had termed 'waves of translation,' forming what were called 'rollers,' or, when on a smaller scale, 'ground swell.' These were a much more troublesome class of waves. It was mainly with them that the engineer had to deal in places open to the Atlantic.

"To reflect or send back the roller was the most effectual plan. For this purpose nothing more was necessary than a deep perpendicular face of perfect masonry, and so long as it stood firm it was faultless, for the reflection really converted the whole effect of the roller on itself into a simple pressure of water. When such a wave was reflected on a perpendicular wall, it merely provided a hydraulic pressure equal to that due to little more than double its own height."

Rankine says:

"When waves roll against a vertical wall they are reflected, and the particles of water for a certain distance in front of the wall have motions compounded of those due to the direct and reflected waves. The particles in contact with the wall move up and down through a height equal to double the original height of the wave."

This is in accordance with well-known mechanical laws. The work stored in a wave must necessarily be measured by the weight of water in the wave, multiplied by the mean distance to which this mass of water has been lifted. The height to which each particle is lifted is determined by the centrifugal force due to the velocity of its rotation. The energy of each particle will be measured by its weight and its velocity of rotation, or, in other words, by its weight and the height to which it will be lifted by the centrifugal force.

To destroy such a wave, each particle must be opposed by a force equal and opposite to that which it has. To reflect or throw it back an additional opposing force of the same amount must be added to it.

A column of water, then, double the wave height would measure the maximum force necessary to reflect the particles in the maximum section through the wave crest. Let Fig. 5 represent a long or solitary wave of elevation (a wave of translation).

$O\ O$ = level of undisturbed water.

a = elevation of wave crest above sea level = $\frac{1}{2} D$.

h = depth of water.

V = velocity of propagation of the wave.

v = uniform horizontal velocity of water in the section $A\ B$.

Then $V : v = h : D$ or $v = \frac{V\ D}{h}$, but $V = \sqrt{g\ h}$,

therefore $v = D \sqrt{\frac{g}{h}}$.

The vis viva of all the particles in the section $A\ B$ will be:

$\frac{1}{2} M v^2 = \frac{M g D^2}{2 h}$ where M is the mass of the particles in the section $A\ B$.

$M g = W = \gamma h$ = weight of particles in section $A\ B$.

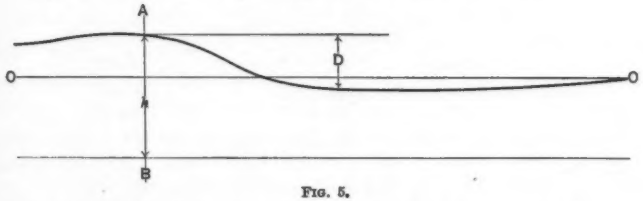


FIG. 5.

Substituting in above, vis viva = $\frac{1}{2} \gamma D^2$; but γD is the weight of particles for a depth equal to the wave height. Hence, the vis viva of such a section, $A\ B$, of a long wave, where the horizontal velocity is uniform for the full depth of the water, is measured by the product of the weight of water γD of the wave section and its mean height $\frac{1}{2} D$, which is identical with that of a column of water of this height. To destroy suddenly the energy of such a wave, it is therefore necessary to present an obstruction capable of resisting a head of water double that of the wave height.

From the practical side, there is a mass of facts and experiments upon the force exerted by the sea which is very impressive and tends to create doubt in regard to the correctness of theoretical deductions. While theories should be tested and judged by a comparison of their

deductions with observed facts and the results of experiments, these latter should be carefully examined and received with much caution when they appear to contradict well-proved mechanical laws. Before questioning the correctness of the theoretical deductions, it would be well to examine the observations and experiments, to see if there are not other elements to be considered, which would explain the phenomena in accordance with true mechanical principles.

The text-books present as evidence of the enormous power of the waves the experiments of Mr. Thomas Stevenson by means of a dynamometer, the transportation of large masses of rock or masonry, the quarrying of rock from its bed, the great elevations to which spray or even solid water is thrown by the breaking of waves against cliffs or artificial structures, etc. An examination of the recorded facts should be made to see if proper and full consideration has been given to the various elements effecting the interpretation of such phenomena.

Stevenson by means of his marine dynamometer obtained at Skerrymore Rocks in the Atlantic Ocean the following average and maximum results in pounds pressure per square foot of surface exposed.

Average of summer months, 1843 and 1844....	611 lbs.
" winter " "	2 086 "
Maximum, March 29th, 1845.....	6 083 "

In the German Ocean he obtained the following pressures per square foot:

Maximum at Bell Rock.....	3 013 lbs.
" Dunbar.....	7 840 "

The dynamometers had disks from 3 to 9 ins. in diameter, but generally 6 ins.; the strength of the springs varied from 10 to 50 lbs. for every $\frac{1}{4}$ in. of elongation.

The pressures obtained by Stevenson are constantly used and accepted as evidence of the force of the waves, and even so careful and reliable an author as Rankine gives them as such, neglecting or overlooking the very important qualifications made by Mr. Stevenson in regard to them.

It is well to note here that Rankine apparently is unwilling to accept these high pressures on any other theory than that the waves did rise high enough to produce such high hydrostatic pressures, for

he says, "the concentration of energy upon small masses of water produces waves of heights greatly exceeding those which occur in water of uniform depth, as the following examples show"; he then tabulates opposite the above pressures the feet of water which would produce them.

The following quotations show Stevenson's own opinion as to the applicability of these pressures as the measure of the wave forces. Referring to his observations, he says:

"It will be sufficient here to state generally the results obtained, only premising that the values refer to areas of limited extent, and are applicable therefore only to the *piece-meal* destruction of masonry, and must not be held as applicable to large surfaces of masonry.

"The experiments at Dunbar and Buckie prove that the sea may exert a force so great as 3½ tons *over the limited extent of surface presented by the disks*, and that the force varies much with the level at which the instruments are set."

The emphasis in all quotations is that of the author quoted.

Clearly, therefore, Stevenson does not accept these dynamometer pressures as the measure of the force of a wave, but solely as local pressures which can be produced by the waves. He gives the following records of great heights to which spray and solid water have been thrown by the action of the sea:

"On February 15th, 1853, during a gale a large body of water was thrown upon the lantern of Nosshead Lighthouse at a height of 175 ft. above the sea.

"At Unst, the most northerly of the Zetland Islands, a door was broken open at a height of 195 ft. above the sea.

"In November, 1827, during a ground swell *without* wind, the water rose to the gilded ball of the Bell Rock lantern 106 ft. It therefore follows that there is a force in action at the foot of the Bell Rock tower competent during ground swells when *there is no wind* to project a column of water to the height of 106 ft., which, according to the laws of hydrodynamics, is due to a pressure of very nearly 3 tons per square foot; whereas, the greatest force that happened to have been actually measured was only 1½ tons.

"It may be added that the spray on an average of 17 observations taken roughly was found to rise on a hollow curved wall about seven times higher than the waves which projected it; and on a vertical wall, taking a mean of 23 observations, it rose 6.6 times."

Now, if it had happened that the water which was thrown to those great heights had been thrown against the surfaces of the disks, the dyna-

mometers, as Mr. Stevenson suggests, would have recorded the much higher pressures due to the height of those columns of water. Such pressures, however, would not measure the force of the waves, for only a small portion of the water in the wave was projected to those heights.

That the energy of a wave, upon meeting an obstacle like a wall or tower, would be expended in giving a greater velocity to those particles near the free surface and in projecting them in the direction of the least resistance, is not mysterious. The action of a hydraulic ram in lifting water higher than the head of the operating current is precisely similar. To determine the energy of such a wave, it is necessary to know not only the number of particles that are raised and their velocities, but also the greatest number of particles that are finally stopped.

That such a concentration of energy into a small portion of the water can cause very high local pressures, which should be considered in the detail of the work is unquestioned, but these pressures cannot be taken as the measure of the effect of a wave upon the stability of a large mass of masonry. They would bear about the same relation to the action of a wave, considered as a whole, as the local action of an engine wheel in crushing or indenting a rail would to the action of an engine or train load upon a bridge structure.

In addition to the records of the dynamometer and of the heights to which water has been thrown, other evidence as to the immense power of the sea has been presented, consisting of the movement of heavy blocks of stone, the breaking out or quarrying of rock from its bed, the breaking of timber booms, etc. The conclusions drawn from many of these instances, where sufficient data have been furnished as a basis of judgment, are, to the author's mind, entirely fallacious.

The cautious investigator would put little reliance on a calculation of the force of a wave from the breaking of wet timbers by repeated blows. The breaking out from their beds of blocks of stone or rock in place by the impact of a wave upon water already contained in the seams is a well-recognized process which in no manner conveys any knowledge of the force of the wave. The movement of masses of rock 6 to 13 tons in weight from their original beds 70 to 75 ft. above the sea level, as noted by Mr. Stevenson at Whalsey Skerries, Shetland, is very impressive. The action, however, is not mysterious, when the immense waves which occur at such places are considered. At Round Island, Scilly Group, large masses of water have been driven over the

summit of this rocky island, which is 130 ft. in height, and small 2½-ounce stones carried to the top of a dwelling house 143 ft. above high water. At Bishop Rock Lighthouse, about 5 miles from the island, waves 50 ft. above high water have been noted. These waves would probably be about 66⅔ ft. from crest to trough. That waves of this size breaking upon a sloping rock or against a lighthouse should produce the destructive effects recorded is not astonishing, but that writers on wave force should assume these extraordinary results as some inherent force due to waves regardless of their height is surprising.

The author, while accepting all these evidences of the force of the sea at various localities, desires to show that they are due to the size of the waves at each locality, and, therefore, when the actual maximum waves are known, it is possible to estimate the resistance which must be given to every structure to meet the force of such waves.

Mr. Stevenson, after enumerating many wonderful effects of sea waves, all within his own personal observations, says:

"We should never expect that examples of the development of the *greatest* force would be found to be against masonry of those artificial works which form our ports and harbors. Accordingly, the examples of the most violent wave action which have been heretofore mentioned were cases of the dislocation or movement of dislocated natural rocks.

"This, however, no longer holds true. The most startling example now on record is that of an artificial work."

He then describes the Wicks Breakwater and the struggle to maintain it against the sea. For a full report of this work see Mr. Stevenson's reports. It will be sufficient for the present purpose to make the following extracts:

"In December, 1872, an enormous mass of concrete, at the end of the work, weighing not less than 1 350 tons, succumbed to the force of the sea, and the resident engineer actually saw it from adjacent cliffs being gradually 'slewed' round by successive strokes, until it was finally removed and deposited inside of the pier, having sustained no damage but a slight fracture of the edges. It was composed of three courses of large blocks of 80 to 100 tons (see Fig. 6), which were deposited as a foundation on the rubble. Above this foundation there were three courses of large stones carefully set in cement, and the whole was surmounted by a large monolith of cement rubble measuring about 26 ft. by 45 ft. by 11 ft. in thickness, and at 16 ft. to the ton, weighing upwards of 800 tons. This block was built *in situ*. As a

further precaution, iron rods 3¼ ins. in diameter were fixed in the uppermost of the foundation courses of cement rubble. These rods were carried through the courses of stonework by holes cut in the stone, and were finally embedded in the monolithic mass which formed the upper portion of the pier."

The shaded portion of Fig. 6 shows the part removed by the waves. This mass weighed no less than 1 350 tons, and presented an area of about 496 sq. ft. to the sea. The lower or foundation course of 80-ton blocks, *b b*, retained their positions unmoved. The second course of cement blocks, *a a*, on which the 1 350 tons rested, were swept off after being relieved of the superincumbent weight, and some of them were found entire near the end of the breakwater.

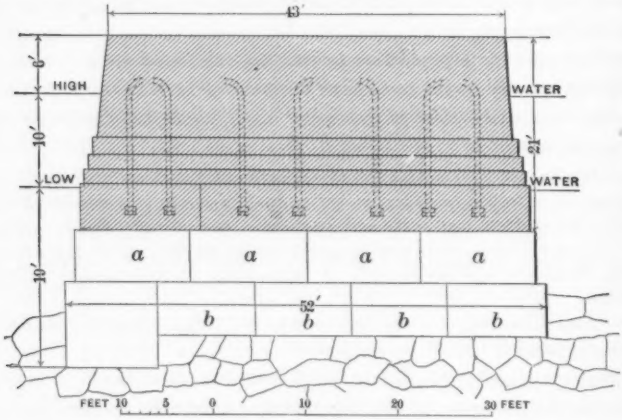


FIG. 6.

The waves by which the works had on various occasions been assailed, and which had carried away the outer portion of the work, were estimated by the resident engineer at 42 ft. from crest to hollow. They passed over the top of the parapet in masses of solid water estimated to be from 25 to 30 ft. deep, and, as ascertained by photographic views, the clouds of spray were projected to a height of not less than 150 ft. If it is assumed, as seems perfectly justifiable, that the horizontal bed joints along the top of blocks *a a*, which were about 5 ft. below low water, were not water-tight joints, the moving of this great mass by the waves of the above description does not appear so extraordinary. The buoyancy of the water would reduce the effective

weight of this block when submerged in the wave to about 730 tons. With a frictional resistance on its bed as high as 75%, a pressure per square foot of surface of about 2 500 lbs. would be sufficient to move the block. The maximum pressure exerted by a wave 42 ft. high would be theoretically about 5 400 lbs. per square foot. It does not appear necessary, therefore, to assume any more mysterious force possessed by such a wave than would be expected by theoretical considerations, to explain its action in this case. In fact, as was afterwards the case, it would not be surprising to find even a larger mass of masonry moved by it.

In 1877 another concrete mass containing 1 500 cu. yds. of cement rubble and weighing about 2 600 tons, substituted for the one just described, was in like manner carried away.

In considering this case a friction of 75% has been assumed for this mass upon its bed. It seems very probable to the author's mind, however, that the impact of the waves upon the water in the bottom seams would greatly assist the other displacing forces.

In Volume CVIII of the *Proceedings* of the Institution of Civil Engineers, there is a very interesting description of the lighthouses built at Bishop's Rock, Scilly Islands. The first, a skeleton iron lighthouse was entirely swept away February 5th, 1850. This was replaced by a masonry lighthouse in 1858. In a brief time the great effects of the sea were noted. During a severe storm a fog-bell weighing 5 cwt. was torn from its bracket on the lantern gallery, 100 ft. above high water.

"When a hurricane of this type was raging, the tower vibrated violently; articles were shaken from the shelves in the rooms and the prisms of the dioptic apparatus fractured. Later on it was discovered that some of the external blocks of granite situated a few feet above high water had been split by the excessive strain imposed on the building.

"The tower was then strengthened from top to bottom by bolting heavy iron ties to the internal surface of the walls and connecting them through the floors. Nevertheless, after a violent storm in 1881, there was again evidence that the heavy strains of the sea had caused further damage. In certain places pieces of granite weighing ½ cwt. were split from the face of the exterior blocks a few feet above high-water level."

It was therefore resolved to strengthen and improve the tower. As it was apparently on the verge of destruction by the storms to which it

had been subjected, it will be interesting to apply the preceding theoretical views to it. Sir James N. Douglas says:

"After the completion of the second work, they had more detailed information through the light-keepers as to the exact movement of those heavy seas. The primary wave flowed freely over the rock at a height of 50 ft. above its summit. There was very little break in the wave except on the tower itself; and there a rather lighter wave arose and reached the cavetto of the lantern gallery (100 ft. above high water), where a third and still lighter wave was produced of heavy spray, which struck the lantern and rising far above it, was driven a long way to leeward, sometimes nearly $\frac{1}{4}$ mile during a heavy storm. He had visited many parts of the world, east and west, but had never seen heavier seas than he had witnessed at Bishop's Rock, where everything appeared favorable to their development."

This tower was 36 ft. in diameter at the high-water line, and for 50 ft. above high water had an exposed wave surface of 1 320 sq. ft., and above the 50 ft. an exposed wind surface of about 1 275 sq. ft. (dimensions scaled from drawings). The cubic contents of masonry was about 38 000 cu. ft.; the weight of the tower and contents was estimated at 6 500 000 lbs., equal to an insistent pressure of about 6 400 lbs. per square foot on the base.

A wave which rose 50 ft. above the rock, which was about high water, would measure at least $66\frac{2}{3}$ ft. from crest to trough. Such a wave reflected from a flat vertical surface would exert a pressure due to the mean of a water column $133\frac{1}{3}$ ft. high, but $16\frac{2}{3}$ ft. of this would be below the base of tower, so only $116\frac{2}{3}$ ft. will be taken.

For a circular tower, there are theoretical grounds for believing that the pressure will be only one-third of that upon a continuous wall. Then there will be a mean pressure of $116\frac{2}{3} \times 64.3 \div 3 \times 2 = 1\,249$ lbs. per square foot of exposed surface; $1\,249 \times 1\,320$ sq. ft. $= 1\,648\,680$ lbs., which gives an overturning movement of about $1\,648\,680 \times 17 = 28\,027\,560$ foot-pounds.

Forty pounds of wind pressure acting on the upper surface of the tower would increase this moment 2 000 000 foot-pounds, making a total moment of 30 027 560 foot-pounds. The moment of resistance of the base would be $\frac{I}{z} = \frac{\pi \times 18^3}{4} = 4\,580.$

$$\frac{30\,027\,560}{4\,580} = \pm 6\,556 \text{ lbs.}$$

causing a reduction of the pressure on the base at the windward side

from 6 400 lbs. to a tension of 156 lbs., and an increase on the leeward side from 6 400 to 12 956 lbs.

No masonry structure could be expected to stand such conditions of strain for a long time.

If, then, it is possible to explain these extreme cases of sea force upon well-recognized mechanical principles, it is safe to accept these same principles as sufficient for estimating the forces which sea structures will have to encounter. In estimating the forces to be resisted by a sea structure, the greatest waves which will possibly impinge against the structure when built must be determined. These may be entirely different from the off-shore waves. It will be necessary to study all the conditions of the location to determine the probable maximum waves which may have to be encountered, if the maximum waves at that point are not already known.

The off-shore waves running into shallower water or converging into narrower limits by the form of the coast or bottom will impart their energy to smaller masses of water, producing waves exceeding those originating them. Or on the contrary, the character of the bottom or foreshore and depths of water may be such as to cause these off-shore waves to break and lose their energy before reaching the structure.

It may be well to quote further from Mr. Thomas Stevenson:

"The determination of the stability of a practically monolithic mass, such as the Eddystone or Bell Rock lighthouses, though difficult enough, is, however, of a simpler nature than that of the disconnected materials of a harbor work. In the masonry of lighthouses the sea is excluded from the joints of the stones, but in many harbor works, the jet of water enters freely into the interior of the masonry and introduces such complexity into the question as to render it impossible, at least in the present imperfect state of our information, to give any rules for directing the engineer. I can therefore simply point out the nature of the different forces which enter into the question of the stability of such loosely built structures, so as to show in what direction the sea attacks the works.

"The impact of the waves against the outside of a sea wall or pier gives rise to four distinct forces, viz.: First, the direct horizontal force which tends to shake loose or drive in the blocks of which the masonry consists; second, the vertical force acting upwards on any projecting stone or protuberance, as well as against the lying beds of the stones; third, the vertical force acting downwards, which results either from the wave breaking upon the toe of the talus wall or from its passing over the parapet and falling upon the pitching behind, so

as to plough it up ; fourth, the back-draught which tends by reaction from the wall to remove the soft bottom, and in this way to undermine the lower courses of the work."

Again he says :

"Within the masonry, as well as without, the waves exert forces in the following different ways: First, by propulsion of vibrations produced by the shock of the waves on the outer or sea wall through the body of the pier to the inner or quay wall. Second, by the direct communication of the impulses through the particles of the fluid occupying the interstices of the hearting so as to act against the back joints of the face wall of the quay. Third, by the sudden condensations and expansions of the air in the hearting so as to loosen and at last to blow out the face stone of the quay, combined with, fourth, the hydrostatic pressure of the water which is forced through the sea wall and from want of free exit is retained and acts as a head at the back of the quay and which, however small in quantity, will act as in a Bramah press upon all surfaces exposed to its pressure, however great those surfaces may be. The three last causes are probably the most efficient agents in the work of destruction."

DISCUSSION.

CHARLES MACDONALD, M. Am. Soc. C. E.—The effect of waves is well shown in the experience at the breakwater at Havre, France. The exposed part of this breakwater is curved like that of the front of a masonry dam of large size. When the breakwater was finished it was found that the waves passed over the top and into the basin beyond, to the serious inconvenience of vessels. This action was reduced by dumping boulders at the toe of the sea face of the wall. The energy of the waves was spent in rolling the boulders upward. The plan is still followed, a certain amount of stone being dumped into the sea at the front of the wall every year to replace that ground to sand and washed away. Does not this indicate a decided frictional reaction? Mr. Macdonald

CORRESPONDENCE.

RUDOLPH HERING, M. Am. Soc. C. E.—The force of waves acting vertically upward should undoubtedly be given consideration, although in most cases it may be small compared to other forces, and furnish merely a reason for avoiding projections from the face of structures exposed to wave action. Mr. Stevenson, see page 157 of the paper, has considered its action against "any projecting stone or protuberance, as well as against the lying beds of the stones." Mr. Hering.

In speaking of the dynamometer pressures of waves obtained by Mr. Stevenson, the author says that they do not measure the force of a wave, but solely a local pressure which can be produced by it, as only a small portion of the water in the wave is projected to the recorded heights. He therefore thinks that the high pressures should be considered in the details of the work, but questions the necessity of taking such pressures as a measure of the effect upon a large surface of masonry. The writer does not think this conclusion sufficiently definite, nor can he see the identity of the parallel drawn between wave pressure and the pressure of an engine wheel on a bridge. The latter consists of a framework, with different duties assigned to each member, while in a sea wall there is a comparatively uniform, homogeneous and solid mass, with the same cross-section at every part, substantially even when buttresses are added. If every part of a wall is made strong enough to resist these enormous isolated pressures, why, then, should not the whole wall be practically strong enough to resist them, even if extending over a large surface, and to what area of wall would the author have this action applied, when he says that at the Scilly Islands, "large masses of water have been driven over the summit of this rocky island, which is 130 ft. in height"? The press-

Mr. Hering. ure of sea water 130 ft. in height is 8 340 lbs. per square foot, or more than Stevenson's greatest dynamometer measurement on small disks at Dunbar, which was 7 840 lbs. To drive "large masses" of water over such an island means a large mass at the bottom, and to drive water up an inclined rocky slope to such a height requires a still greater force at its foot than that mentioned. It seems necessary to the writer to take up each case of this kind by itself, and to consider it in the light of all forces liable to act, instead of generalizing further than to find out what simple mechanical principles apply.

The writer does not consider the division of free waves into "long" waves, "oscillating" waves and "ripples" a very happy one. Oscillating waves, particularly in the ocean, can be very much longer than the so-called "long" waves in very shallow water, or near the shore. Ripples, the author explains, run by surface tension. So far as the writer's experience goes, ripples are waves generally caused by the friction of air moving over the surface of water, like most other waves. They can be seen plainly when a slight gust of wind starts over a perfectly calm sheet of water, as in a pond. The writer has also seen them under the same conditions on the ocean after a day's calm. They are the first result after wind commences. If the wind continues and blows stronger, then another and larger set of waves is generated, upon the surface of which the ripples still continue, though less regular in form. If the wind blows still stronger, still larger waves set in, carrying the two other kinds of waves on their surface. When crossing the Atlantic, the writer could, on a certain occasion, distinctly discern four classes. On the Jersey shore, perhaps twelve hours after a storm, when watching the surf, he once distinctly noticed that about every seventh to tenth breaker would run further up the beach than the others, and, in addition to the breaking waves, smaller waves or ripples could be seen upon them which had been started by a fresh breeze. It was evident that there still existed the effect of some large waves coming in from the ocean and caused by the previous storm.

On a dry, sandy beach similar effects can also be noticed. What in this case are often called sand ripples are also the effect of the friction of air upon the particles of sand and are not due to surface tension. A flag or streamer waving in a breeze has its wavy motion imparted by the friction of passing air, similar to the case of water waves.

Generally speaking, there seems to be no difference in kind between large and small free waves, but merely a difference of degree. A definite relation exists between the causing force and the character of the material which is set in motion and its freedom to move. The writer had never seen this general problem worked out, but it applies equally to solid, fluid and gaseous bodies moving over each other, and of which there are nine possible cases. Unless the friction is reduced or wholly overcome, as by a lubricant, such movements produce

oscillations. These may be in the form of sound waves of high pitch, as in some cases of solids moving upon solids, or merely a rattling noise when, for instance, the ferrule of an umbrella is made to slip over the surface of a sidewalk, or these oscillations may be a visible tremor as when water flows through a hose, or be in the form of a spiral motion assumed by bubbles of air when rising through water. Mr. Hering.

Unless the author can offer a better explanation, it seems to the writer that "surface tension" rather explains the somewhat remarkable fact, that oil poured upon highly agitated water will, by hanging together and spreading out into a thin film, actually reduce the oscillation of waves and completely stop the ripples. The water surface does then not as readily yield to the attack of the wind, nor permit the breaking of the waves, the so-called whitecaps. It is trustworthily stated that a teaspoonful of oil will "calm" a surface of 1 000 square meters or about ¼ acre on the ocean. Heavy and thick oils are said to be the most effective.

ROBERT FLETCHER, Assoc. Am. Soc. C. E.—Among the earlier contributions to the literature of civil engineering is an interesting work by Col. A. R. Émy, an officer of the corps of engineers of the French army, entitled: "Du Mouvement des Ondes et des Travaux Hydrauliques Maritimes, Paris, 1831." The author devotes considerable space to disproving the then commonly accepted theory that in wave motion the individual particles of water move up and down by filaments which, through all phases of the displacement, remain vertical. Having shown this theory to be untenable, he then opposes the practice of giving to exposed sea-walls or breakwaters a vertical or slightly battered face, bringing to bear a great array of facts and arguments to show that such structures are assailed by enormous masses of water acting by horizontal impact, and should deflect rather than reflect such masses. Mr. Fletcher.

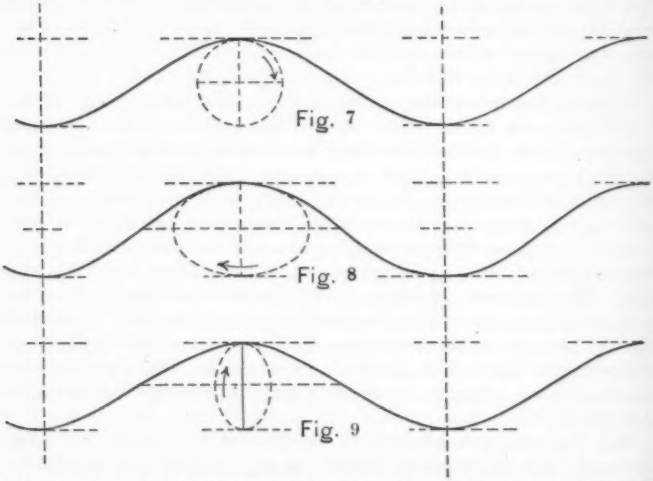
Col. Émy was recognized by one or two of his contemporaries as among the first to propound the true theory of wave motion, that is, the orbital movement of the individual particles. His treatment of the subject is free from mathematical analysis, although he presents very clear and ingenious geometrical demonstrations; he argues from facts gathered by his own long-continued observations of wave action and some of its results, while he appeals to the common phenomena with which even less diligent observers are familiar.

Figs, 7, 8 and 9 are drawn on the assumption that the velocity of the particle in the orbit is constant during a complete oscillation, but with the depth of trough exaggerated in proportion to the length of wave. Émy claims that the orbit cannot be circular because the wave profile would then be a curve of sines, and the most cursory observation shows that this is not the fact. He claimed also that the elliptical orbit with minor axis vertical would give a form equally at vari-

Mr. Fletcher. ance with actual appearance; but that the elliptical orbit with major axis vertical accounts for all actual circumstances of form and motion.

That pure wave motion in deep mid-ocean consists in the motion of the individual particles in elliptical orbits, and that the size of orbit decreases as the depth increases, until perfect calm exists, is now the commonly accepted theory. Émy laid chief emphasis, however, on his conception and definition of the term *flot de fond* or "bottom wave." He says: "I have discovered that the true bottom waves constitute a very important phenomenon of which the nature has not hitherto been recognized, although some of its effects have been noted."

When waves approach the shore from the ideal position in deep mid-ocean and feel the influence of bottom and projecting lands, their



behavior is no longer fully explained by the purely orbital motion of the molecules of water. The theory advanced is chiefly concerned with the effects that the bottom exercises upon the waves. This influence of the bottom is claimed to be of three kinds.

First.—When the general depth is small and the waves are large, they will be shortened and raised, having a more pointed shape due to the reaction of the bottom, causing the vertical axis of the ellipse to be lengthened.

Second.—If the general depth of the sea or ocean is great, and diminishes towards the shore by very gentle and uniform slopes, the undulations are destroyed by gradually stopping the lower particles in their orbits, and also modifying the undulations of the surface particles.

Third.—The third action is caused, in the ideal and extreme case, by sudden rises or steps in the bottom, and is of the greatest importance. Referring to Fig. 10, let *O R* represent the surface of the calm sea, the curve *B C D*, etc., a vertical section of the wave surface, and *P Q* the level where no movement exists, so that the bottom below this level has no influence upon the wave motion. Let *U T* be a sharp rise, followed by the nearly horizontal bottom *TS*. The undulation *b c d e*, etc., is tangent to *TS*, and it is evident that all the particles of water above this curve will have sufficient space in which to describe their orbits, but that all particles below the curve will lack the room required for a full undulation. The latter particles form under each wave of the undulating mass bodies of water limited by the curve and the Mr. Fletcher.

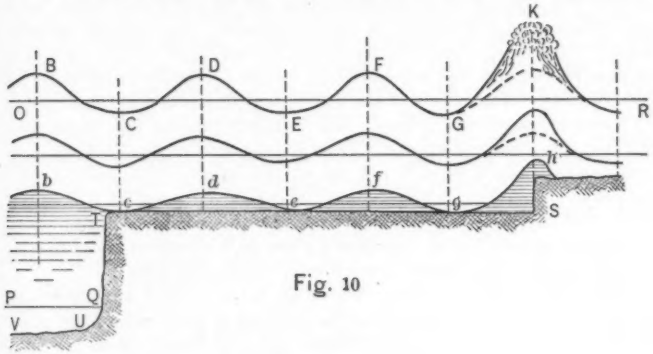


Fig. 10

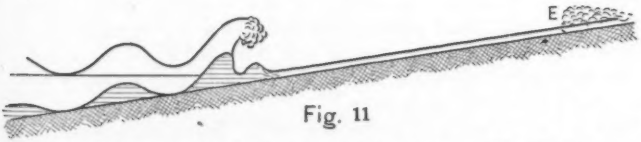


Fig. 11

bottom. These entrained masses of water with profiles of the general shape *c d e* are subject to pressure on the rear, for the wave above is descending to form the trough, while it is relatively relieved from pressure in front, for the water above is rising to form the crest of the advancing wave. By the purely oscillatory action of the wave above, or surface wave, the entrained mass must travel in the direction of the wave motion and with the same velocity. Each oscillatory and apparently advancing wave* entrains a new body of water, and these masses

* In the normal or oscillatory wave no particle of water has a greater horizontal displacement than the length of the horizontal axis of its orbit. Any person who has attentively observed a floating object tossed by waves where there is no current has recognized this as an elementary and fundamental fact. It is illustrated by the waves which sweep over a field of ripened grain and by familiar experiments in the physical laboratory.

Mr. Fletcher.

of water, forced along the bottom under the normal waves, are defined as "bottom waves." Here is a true wave of translation, which has little or nothing of an oscillatory character, and which must exert horizontal energy nearly in proportion to its mass and the square of its horizontal velocity.

When the bottom rises in a series of slight but abrupt ascents, the bottom wave formed at the first rise is forced to mount those succeeding by the pressure from the wave above it, and, as each rise causes a bottom wave on its own account, the bottom waves increase in volume as they proceed, and may carry a large mass of water towards the shore. If such waves meet an inclined plane after passing the sharp rises, they are retarded by the bottom, and, as the pressure continues with little diminution, they are forced into shorter and higher waves, which cause the surface waves to change in form similarly. The energy required to give the bottom waves their velocity is furnished by the normal or surface waves, and this loss of energy causes the latter to shorten and lose velocity. The effects of the incline may be likened to those caused by a succession of small rises. Fig. 11 may represent the appearance at the instant when the bottom wave has become of such a form as to break the surface wave and run up on the shore. The sudden projection of the surf upon the inclined shore is indicated at *E*, Fig. 11. If the velocity and volume of the bottom wave are great, it may break the surface wave when it encounters a rise in the bottom, as at *K*, Fig. 10, causing foam and spray as each wave passes this point.

Often this breaking of the waves over a submerged bank destroys the undulation to such an extent that the sea is quite calm after passing the line of the surf. The pressure upon the bottom wave may not be sufficient to bring any part of it near the surface; but its influence will always be shown by some change in the character of the surface waves, since such a fluid mass is very sensitive to any expenditure of energy. The bottom waves, formed and increased in volume by successive shoaling of the water, are hurled against steep shores or artificial breakwaters with great force.

Such is Émy's theory of the bottom wave. Being literally outside of the limits of the mathematical wave of oscillation, its existence and action might not be considered demonstrable by pure analysis, but the hypothesis does not contravene physical laws. Later authors, English and Italian, cite observations and draw conclusions as to the proper profile of sea-wall sections which are in exact accordance with the doctrine and practice which Émy developed from this theory. It is certain that any mathematical theory of wave action rests upon ideal conditions quite as extreme and unusual as those upon which the theory of the bottom wave is based. The simple, unmodified undulation of the surface of the sea, showing the ideal billow, is not to be seen every day. The great swell of the ocean presents it on a large scale, but

Mr. Fletcher.

usually there are primary, secondary and tertiary systems, one superimposed on the other, one from distant and another from local sources of disturbance, with modifications, so that the resultant action is often apparently confused, irregular and interrupted. Hence the engineer desiring a proper working knowledge as to the principles of construction of maritime works must study chiefly facts, phenomena and the accumulated experience of his predecessors in this line. Some facts and phenomena in addition to those cited in the paper will now be presented.

Prof. Airy[1] asserts that the horizontal motion of the particles of water next to the bottom, produced in shoal water by long waves, is proved by the sensible disturbance of the sand and stones of the bottom; and that waves break over the edge of the Bank of Newfoundland when the waves are high, although the depth of water on the shallow side of the bank is 500 ft. He also mentions that off the coast of the British Isles, where the best charts show a sudden falling off of the bottom, a similar breaking of the waves occurs.

Similar action is noted by Mr. Kiddle[2]. Speaking of the general belief that the influence of heavy waves ceases at a few fathoms beneath the surface, where all is quiet, he says that swells show marked increase in height when they roll over the edge of soundings. "On the parallel of Cape Clear, in longitude 15° W., seamen are familiar with this phenomenon, although the depth is nearly 500 fathoms. At times it is so marked that the dead reckoning may be checked by carefully noting the increase in the depths of the hollows of the waves." "Shortly after the edge of the sounding is passed, the sea becomes more regular and less dangerous to vessels."

The great depth to which the action of waves may be felt is illustrated by a common occurrence off Nantucket Shoals, noted by Mr. Kiddle. Pilots and masters of vessels assert that sand is frequently left on deck after a sea has broken on board, although the depth of water is from 75 to 90 ft. The waves of the Pacific off the western coast of the United States are described by observers as much greater than those of the Atlantic, and many record their breaking in 15 fathoms of water.[3] Charts[4] show the bottom to be in general more precipitous than the bottom off the eastern coast.

In the opinion of Mr. Michael Scott[5]:

When waves come upon a slope the vertical oscillation is converted into a motion parallel to the slope. Surf waves are greatest when deep water shoals suddenly. When the water shoals the waves rear up, change their profile and have a progressive motion imparted to them.

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[1] "Encyclopedia Metropolitana." Article, "Tides and Waves."

[2] "Nature," December 9th, 1875.

[3] Report of Chief of Engineers, U. S. Army, 1881, p. 2635.

[4] U. S. Coast Survey Charts, Nos. 676 and 681.

[5] *Proceedings* of the Institution of Civil Engineers, Vol. xviii, pp. 75, 76.

Mr. Fletcher. Mr. Cockburn Curtis, who had extensive practice on coast surveys under the British Admiralty in all parts of the world, in a discussion on the form of breakwaters,[1] said that much might be done toward determining the best form and materials for their construction by a careful observation of the character of the waves and of the form of bottom over which they approach the shore. He found that when the general configuration was decidedly steep at the shore line, there was very little change in the waves; but where great differences of level, or steps, existed in the profile, what might be called shoal waves or surf waves were generated. Mr. Curtis decided from his study of the surf waves on the west coast of Central America and on the coast of Coromandel, where he believed the heaviest surf to exist, that the waves were generally greatest where, in approaching the shore, the deep water was suddenly succeeded by a long series of shoals. "The ocean or deep-water wave, when it came in contact with the first shoal water, immediately appeared to rear up or increase in height and subsequently to alter its profile so that the mass of the water was thrown into the front part of the wave and a progressive motion was communicated to the particles of water. * * * The consequence was that the wave always took up with it a mass of water along the slope."

The author of an article entitled, "Notes on the Effects of the Waves Upon the Neapolitan Shore, in Connection with the New Coast Works," gives[2] the following conclusions from his observations:

"Whatever theory may be entertained as to the origin of waves, this fact is undisputed, that near the shore the vertical motion is transformed, as the depth diminishes, into a more or less rapid horizontal movement of translation, and that, by reason of this transformation, the waves acquire a horizontal action upon shallow shores which at times is very energetic, causing them to break violently against any obstacle they may encounter. The translation may be partial or complete; in the first case, they are 'bottom waves'; in the second the water has become shallow enough to cause the 'bottom wave' to be the greater part of the whole wave, which causes the entire mass of water to be thrown forward."

The following facts are well known: When a body of slightly greater specific gravity than the water is thrown off from the beach it sinks, but is returned to the shore very soon. Debris and portions of the cargo from wrecked vessels, whether capable of floating or not, are driven ashore. Dead bodies are washed up in the same manner (by the bottom waves?). A remarkable instance of this action is given by Sir John Robinson.[3] At Madras, during a violent storm a quantity of pig lead was cast on the beach, and it was proved that it came from a vessel which had been wrecked more than a mile off shore.

Mr. John Murray,[4] when working in a diving bell at Sunderland, in 20 ft. of water, found it impossible to work on one occasion because

[1] *Proceedings* of the Institution of Civil Engineers, Vol. vi, p. 125.

[2] *L'Ingegneria Civile e l'Arti Industriali*, 1885, p. 53.

[3] *Proceedings* of the Institution of Civil Engineers, Vol. ii, p. 128.

[4] *Proceedings* of the Institution of Civil Engineers, Vol. xix, page 669.

of the great agitation of the water at the bottom, although at the surface there was but a slight swell. This is a direct proof of the movement of translation at the bottom, while the surface presented only the movement of oscillation. Mr. Fletcher.

Smeaton's account of the tremendous force of the waves at Eddystone reads:[1] "The waves would strike the rock and house and fly up in a white column, enveloping it like a sheet, rising to at least twice its height (about 200 ft.) and totally intercepting it from sight." Smeaton believed the cause of this action to be due to the position of the reefs and the shape of the bottom rock. "A circumstance which tends to augment the force and height of the sea so breaking upon these rocks is their particular form and position. * * * They not only stretch across the channel in a north and south position to the length of about 100 fathoms; but they lie in a sloping manner towards the southwest." This sloping continues below the water for a mile or more. "From this configuration it happens that seas coming uncontrolled from the deep water, and, rather suddenly at last, though gradually meeting the slope of the rocky bottom, they are swelled to a degree, in storms and hard gales of wind, as to break upon the rocks with most dreadful violence." The chart of the locality shows that the reefs form a gradually rising trough or gorge, doubtless broken by steps and ridges, leading up to the level where the stone tower was built, from the direction of the heaviest seas. "I particularly noticed the manner in which the waves began to gather, as soon as they came so near the house as to be sensible to the sloping rocks underneath them. These waves, by degrees towering higher as they came nearer, formed a deep hollow sea at the foot of the building; and then, falling into it, struck it with all imaginable fury."

The late Thomas Stevenson, referring to the wave action at Bell Rock Lighthouse, quoted the late Robert Stevenson as follows:[2] "The dispersion of fishes, by their disappearance from the fishing grounds in stormy weather, tends to show the disturbance of the depth of the ocean at the depth of 30 or 40 fathoms. * * * Some drift stones of large dimensions, measuring upwards of 30 cu. ft., or more than 2 tons in weight, have, during storms, been thrown up from deep waters. These stones are so familiar to the lighthouse keepers at this station as to be by them termed travelers."

It appears from these citations that a motion of translation of portions of the wave when it reaches shallow water is now generally admitted; also that where abrupt rises in the bottom are known to exist, there the phenomena are observed which Émy's theory describes and explains.

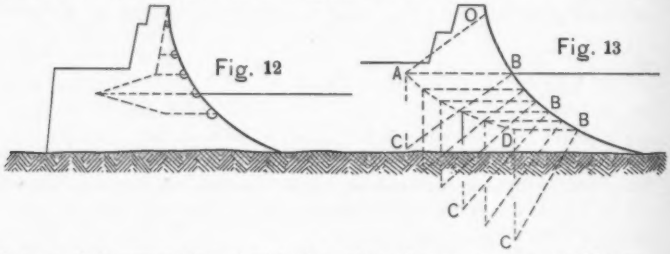
Thomas Stevenson had no records of pressures near the bottom. At Skerryvore Rocks the high and low dynamometers were not placed

[1] John Smeaton's "Narrative of the Building and a Description of the Construction of the Eddystone Lighthouse," pp. 10 and 175.

[2] "Design and Construction of Harbors," Edinburgh, 1874, p. 17.

Mr. Fletcher. upon the same rock, but one was 40 ft. seaward of the other, and hence subject to different conditions. At Dunbar flush dynamometers were placed over quite an extent of the curved wall, as shown in Fig. 12. The dotted lines show the variation of horizontal pressure thought to exist. Stevenson gave the diagram as a rough approximation only, as some of the instruments failed.

The following argument is by Mr. A. W. French, the writer's colleague: Masses of water hurled against the foot of a curved wall would be deflected from their horizontal course and would exercise a nearly constant normal pressure on the surface. Of course the horizontal pressure would increase as it passed up the curve. In Fig. 13, with a profile similar to that of Fig. 12, the change in horizontal pressure due to the reaction of deflection, or centrifugal force $B\,C = \frac{W\,v^2}{g\,R}$ (w is the weight of a cubic foot of the water, and R is the radius of curvature) is represented by the broken line $A\,D\,B$. The energy of the moving and lifted water would be expended against the force of gravity and would become zero at some point on or above the wall. Some



line, as $A\,O$, connecting A with the point of zero energy, would complete the line of pressure.

The similarity of Figs. 12 and 13 is apparent. Stevenson's figure shows a sharper diagram, but it is reasonable to suppose that this is due to the impulse of the surface particles as they are suddenly stopped in their orbits by the wall. This impulse would lengthen the line $A\,B$ and make the agreement still closer.

At Dunbar the action upon the unfinished wall was noteworthy. Dynamometers were attached to piles a few feet in front of the wall and also on the wall itself and pressures noted. Stevenson remarks:

"These experiments prove, therefore, that oscillatory waves become waves of translation when they reach the unfinished part of a vertical sea wall, and that they then exert a force nearly six times greater than if they had remained waves of oscillation. It further appears that purely oscillatory waves do not exert much more than their hydrostatic pressure, under conditions similar to those affecting the Dunbar experiments; but had there been a storm of wind, those waves would no doubt have ceased to be purely oscillatory, even though the water had been deep."

Mr. Fletcher. Referring to the Skerryvore experiments, assume the maximum dimensions of waves to be 600 ft. in length, 43 ft. in height, with a velocity of 50 ft. per second.[1] The velocity[2] of the surface particles in their orbits would be approximately 12 ft. per second.

$$\frac{\text{Mean speed of particle}}{\text{speed of wave}} = \frac{\text{Circumference of orbit}}{\text{length of wave}}$$

The pressure of the water striking the dynamometer at this velocity would be about 290 lbs. per square foot $\left(p = \frac{W v^2}{g}\right)$. This pressure would decrease from the surface downward. The static pressure of the wave at the sea level would be $43 \times 64.4 = 2\,769$ lbs. per square foot, or an average pressure of 1 385 lbs. If the average pressure due to the impulse of the particles be $290 \div 2 = 145$ lbs., the combined pressure would be only 1 530 lbs. per square foot. But the maximum force recorded was above 6 000 lbs. If, now, some of the water is assumed to move horizontally with the wave velocity of 50 ft. per second, then the force of the impact will be theoretically $\frac{64.4 \times (50)^2}{32.2} =$ 5 000 lbs.; if 1 385 lbs. (neglecting 145 lbs., since the particles of the bottom wave have no orbital motion) is added to this, there results 6 385 lbs., which is a good agreement with the experimental results.

Mr. Cooper. THEODORE COOPER, M. Am. Soc. C. E.—No modern authority on waves holds to that part of Col. Émy's theory which requires that the major axis of the ellipse be vertical. The special cases of wave action mentioned by Prof. Fletcher and the phenomena, for the explanation of which the bottom wave theory was advanced, are all covered sufficiently for the purpose of this paper by the general statement, "the off-shore waves running into shallower water, or converging into narrower limits by the form of the coast or bottom, will impart their energy to smaller masses of water, producing waves exceeding those originating them." While the manner in which waves are formed and modified and their action under various conditions are instructive and full of interest, the important points, from an engineering point of view, are the character of the waves that will reach any proposed structure and the force they will exert against the work. The height of any wave, regardless of the manner of its generation, is the measure of its energy, and therefore the measure of the force necessary to resist such wave, according to the method to be employed, whether to reflect, divert or destroy it.

The computations at the end of Mr. Fletcher's remarks are incorrect. The orbital energy and the static pressure cannot be added, for the static pressure is produced by the orbital energy. Neither can the impact of a wave be determined by use of an impact formula, nor is

[1] Stevenson's "Design and Construction of Harbors," p. 35.
Rankine's "Civil Engineering," p. 754.

Mr. Cooper. it necessary, for the height of the wave measures its energy, and twice this height gives the greatest measure of the pressure needed to reflect it. His assumption that some of the water may move with a velocity equal to the wave velocity is untenable, for such particles could not continue in the wave, but must fly off by centrifugal force. If such an assumption were applied to the Atlantic tidal wave, there would be twice in 24 hours' pressures along the whole seaboard of 1 500 000 lbs. per square foot.

Mr. Hering asks: "If every part of a wall is made strong enough to resist these enormous pressures, why, then, should not the whole wall be practically strong enough to resist them"? Walls are not the only form of sea structures. In a solid wall the joints and projections would be the parts most seriously effected by these enormous pressures acting over only limited areas. In timber crib walls or bulkheads the detail parts might be entirely too weak to resist the localized pressures, and yet the whole crib, unless destroyed in detail, be abundant to resist the onslaught of the waves. Lighthouses which were perfectly secure as to the main structure could be and have been seriously effected in their upper works by these limited but heavy forces.

Mr. Hering does not think the division of waves as accepted by modern writers a happy one, and especially objects to "ripples" or that class of small waves which run mainly by surface tension. Surface tension enters into all the problems of liquids, but usually is a very minor factor. In large waves its effects are unimportant, but for minute waves it becomes the principal factor and modifies and in some cases reverses the laws governing large waves. Ripples run the faster the shorter they are, while waves do the reverse of this.

Any one who desires to study into this interesting subject will find "The Construction of Harbors," by Thomas Stevenson; Tides and Waves in the "Encyclopedia Metropolitana"; "Waves and Wave Theory," in the "Encyclopedia Britannica," good material to commence on. All of these have been freely drawn upon in preparation of the foregoing paper.